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A novel approach to compare the spectral densities of some uncorrelated cyclostationary time series

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Abstract Our primary objective in this article is to compare the spectral densities of some cyclostationary time series. By using the limiting distributions of the discrete Fourier transform, a novel approach is introduced to determine whether the spectral densities of some uncorrelated cyclostationary time series are the same or not. Also, the ability of the proposed technique is examined by employing simulated and real datasets.

1. Introduction

Comparison of several processes is a main subject in economics, physics, chemistry, signal processing and hydrology. Really, the researchers try to compare the stochastic mechanism of some observed datasets from different time series.

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The comparison, classification, regimentation and clustering of two or some time series have been investigated in different time-domain and frequency-domain methods by several scientists. The References [1–14] can be utilized to the stationary processes or the non-stationary processes which are transformable to stationary processes by using differencing. Nevertheless, these approaches can not be applied in numerous situations that we are facing with non-stationary processes. The References [15–16] can be utilized to the processes with fractional Brownian motion noises and almost cyclostationary processes.

Cyclostationary (CS) processes that are presented by Gladyshev [17] are nicely applied to illustrate rhythmic processes. The CS processes (may be also called periodically correlated in statistics) are a big time series group with cyclic mean and auto-covariance functions. These periodicities can not be eliminated by transformations such as differencing. These processes can be fitted on real datasets of different scientific fields, like economics, physics, chemistry, signal processing and materials [18]. The References [19–21] are suitable references about the applications as well as theories of CS processes. Hurd and Gerr [22] considered the detection of periodicity by using two graphical approaches based on the coherency and incoherency statistics. Broszkiewicz-Suwaj [23] applied the bootstrap methodology and constructed a measure of fitness statistic (called MoF, in short) to detect the periodicity. Nematollahi et al. [24] used periodogram asymptotic distribution to establish a goodness of fit test for CS time series. Mahmoudi and Maleki [25] studied the detection of periodicity using the estimating of the spectral support of a given dataset.

In present research, the limiting distribution of the discrete Fourier transform is employed to introduce a new method to investigate the equality of spectral mechanisms of several CS processes. To examine the ability and performance of the proposed technique, numerous simulation study and real dataset are also given. Section 2 is devoted to the notations and preliminaries. The methodology to compare the CS models is given in Section 3. Sections 4 and 5 report the results of simulation study and real world dataset to investigate the ability of the introduced approach.

2. Preliminaries

Definition 1:. *A time series $\{X_t, t \in \mathbb{Z}\}$, is CS with period T (CS-T), if*

$$m(t) := E(X_t) = m(t+T),$$

and

$$R(s,t) := Cov(X_s, X_t) = E\left[(X_s - m(s))\overline{(X_t - m(t))}\right] = R(s+T, t+T)$$

for $s, t \in \mathbb{Z}$.

A time CS-T can be expressed as the following spectral representation on $[0, 2\pi)$,

$$X_t = \int_0^{2\pi} e^{itx}\Psi(dx), t \in \mathbb{Z},$$

such that Ψ is a random measure with following property on $[0, 2\pi)$:

$$E\left(\Psi(d\lambda)\overline{\Psi(d\lambda')}\right) = 0, \lambda - \lambda' \neq 2\pi k, k = -T+1, \cdots, T-1, k \neq 0.$$

We can define the spectral distribution of Ψ, by

$$\boldsymbol{F}(d\lambda) = \left[F_{k-j}\left(d\lambda + \frac{2\pi j}{T}\right)\right]_{j,k=0,\cdots,T-1}, \lambda \in \left[0, \frac{2\pi}{T}\right),$$

where

$$F_k(d\lambda) = E\left(\Psi(d\lambda)\overline{\Psi\left(d\lambda + \frac{2\pi k}{T}\right)}\right) = F\left(d\lambda, d\lambda + \frac{2\pi k}{T}\right),$$

$k = -T+1, \cdots, T-1.$

Also the spectral density of Ψ, $\boldsymbol{f} = \left[f_{jk}\right]_{j,k=0,\cdots,T-1}$, is defined by

$$\boldsymbol{f}(\lambda) = \frac{d\boldsymbol{F}}{d\lambda} = \left[f_{k-j}\left(\lambda + \frac{2\pi j}{T}\right)\right]_{j,k=0,\cdots,T-1}, \lambda \in \left[0, \frac{2\pi}{T}\right),$$

such that the density f_k is corresponding to the F_k.

3. Methodology

Let $\{X_t^{(1)}, t = 1, \cdots, n_1\}$, $\{X_t^{(2)}, t = 1, \cdots, n_2\}, \cdots, \{X_t^{(m)}, t = 1, \cdots, n_m\}$, are observations from m independent CS processes with period T.

In many fields the researchers are interested to compare the spectral mechanisms of the processes $X_t^{(1)}$, …, and $X_t^{(m)}$. In other words, they want to test the following hypothesis:

$$H_0 : \boldsymbol{f}_1 = \cdots = \boldsymbol{f}_m$$

such that $\boldsymbol{f}_1, \cdots,$ and$\boldsymbol{f}_m$ are the spectral density matrices of the time series $X_t^{(1)}$, …, and $X_t^{(m)}$, respectively.

Under the rejection of H_0, we conclude that the spectral densities of at least two processes differ, and if H_0 is accepted, consequently there is not significant difference between the spectral densities of the m processes and the stochastic mechanisms of m processes are the same.

Assume $X_0, \cdots, X_{n-1}$ are the observations from X_t. The discrete Fourier transform of these observations is defined by

$$d_X(\lambda) = n^{-1/2}\sum_{t=0}^{n-1} X_t e^{-it\lambda}, \lambda \in [0, 2\pi).$$

The periodogram of CS time series was defined by Soltani and Azimmohseni [35] as

$$\boldsymbol{I}_X^T(\lambda) = \boldsymbol{d}_X^T(\lambda)\boldsymbol{d}_X^{T*}(\lambda),$$

where

$$\boldsymbol{d}_X^T(\lambda) = (d_X(g_1(\lambda)), d_X(g_2(\lambda)) \cdots, d_X(g_T(\lambda)))', \lambda \in \left[0, \frac{2\pi}{T}\right),$$

such that

$$g_k(x) = x + \frac{2\pi(k-1)}{T},$$

for$k = 1, \cdots, T$.

Lemma 3.1:. *Soltani and Azimmohseni* [26]

For a CS-T processes X_t, assume $\boldsymbol{f}(\lambda), \lambda \in [0, 2\pi)$ is continuous. If $\lambda_1 < \cdots < \lambda_J$ are frequencies in$\left[0, \frac{2\pi}{T}\right)$,then

$\widehat{f}(\lambda) := \frac{I_X^T(\lambda)}{2\pi}$(i) is an asymptotical estimator for $f(\lambda), \lambda \in \left[0, \frac{2\pi}{T}\right)$.

$d_X^T(\lambda_j), j = 1, \cdots, J$,(ii) have asymptotical complex normal distributions

$N_T^c\left(0, 2\pi f(\lambda_j)\right)$.

$d_X^T(\lambda_j), j = 1, \cdots, J$,(iii) are asymptotically independent.

$I_X^T(\lambda_j), j = 1, \cdots, J$,(iv) have asymptotical complex Wishart distributions

$W_T^c\left(1, 2\pi f(\lambda_j)\right)$

$I_X^T(\lambda_j), j = 1, \cdots, J$,(v) are asymptotically independent.

For $j = 1, \cdots, J$, assume

$$Y_j^{(k)} = Re\left(d_{X^{(k)}}^T(\lambda_j)\right), k = 1, 2, \cdots, m,$$

and

$$Z_j^{(k)} = Im\left(d_{X^{(k)}}^T(\lambda_j)\right), k = 1, 2, \cdots, m,$$

where $d_{X^{(k)}}^T(\lambda_j)$is $d_X^T(\lambda_j)$ for the population k^{th}.

Corollary 3.1:. *Let*

$$W_j^{(k)} = \left(Y_j^{(k)}, Z_j^{(k)}\right)', k = 1, 2, \cdots, m, j = 1, \cdots, J.$$

Then for $j = 1, \cdots, J$,

(i) $W_j^{(k)}, k = 1, 2, \cdots, m$,are asymptotically independent.
(ii) The asymptotic distribution of $W_j^{(k)}, k = 1, 2, \cdots, m$,is$N_{2T}\left(0, \Sigma_j^{(k)}\right)$, where

$$\Sigma_j^{(k)} = \begin{bmatrix} V_{Y_jY_j}^{(k)} & V_{Y_jZ_j}^{(k)} \\ V_{Z_jY_j}^{(k)} & V_{Z_jZ_j}^{(k)} \end{bmatrix}, V_{AB} = COV(A, B).$$

Proof:. *This is a straight result of previous lemma.*

Consequently,

$$U^{(k)} = \sum_{j=1}^{J} W_j^{(k)}, k = 1, 2, \cdots, m,$$

is asymptotically$N_{2T}\left(0, \Sigma^{(k)}\right)$, such that

$$\Sigma^{(k)} = \Sigma_1^{(k)} + \cdots + \Sigma_J^{(k)}.$$

3.1. Testing problem

As discussed, in practice the researchers are interested to test

$$H_0 : f_1 = \cdots = f_m$$

This hypothesis can be rewritten as

$$H_0 : \Sigma^{(1)} = \Sigma^{(2)} = \cdots = \Sigma^{(m)}$$

As a consequence, the asymptotic distribution of

$$U = \sum_{i=1}^{m} U^{(i)},$$

$N_{2T}(0, \Sigma)$

$$\Sigma = \sum_{i=1}^{m} \Sigma^{(i)}.$$

Therefore the statistic

$$\chi^2 = (U)'(\Sigma)^{-1}(U),$$

is asymptotically distributed as $\chi^2(2T)$.

Therefore, the statistic χ^2 and its asymptotic distribution can be employed to present a suitable test statistic and critical region about H_0. As it can be seen, the parameter Σ is usually unknown. Therefore, firstly this parameter should be estimated. Let

$$S = \frac{\sum_{i=1}^{m}(N_i - 1)\mathbf{S}^{(i)}}{N - m},$$

as the sample pooled covariance matrix, where

$$N = \sum_{i=1}^{m} N_i,$$

$$\mathbf{S}^{(k)} = \sum_{j=1}^{J} \mathbf{S}_j^{(k)},$$

$$\mathbf{S}_j^{(k)} = \begin{bmatrix} \widehat{V}_{Y_jY_j}^{(k)} & \widehat{V}_{Y_jZ_j}^{(k)} \\ \widehat{V}_{Z_jY_j}^{(k)} & \widehat{V}_{Z_jZ_j}^{(k)} \end{bmatrix}$$

and

$$\widehat{V}_{AB} = \widehat{COV}(A, B).$$

Under H_0, S can consistently estimate the parameter Σ. By using the Weak Law of Large Numbers, it can be concluded that

$$\chi^{2^*} = (U)'(\mathbf{S})^{-1}(U),$$

is asymptotically $\chi^2(2T)$. Therefore the hypothesis H_0 is rejected if $\chi^{2^*} > \chi^2_{1-\alpha}(2T)$, where α is size of test.

Remark 1:. *In real problems, we need more samples from* $d_{X^{(k)}}^T$ *(*N_k *samples for population* $k^{th}, k = 1, 2, \cdots, m$*). The bootstrap estimation methods can be applied to reach this aim.*

In this work, the moving block bootstrap methodology (MBB, in short) [27] will be used.

4. Simulation study

In this section, the performance of the proposed technique is examined for simulated datasets. The steps of simulation procedure are as following:

(i) For the first, the second and the third time series, simulate a sample of size n_1, n_2 and n_3, respectively.
(ii) Calculate $d_X^T(\lambda_j), j = 1, \cdots, J$, separately, for simulated samples.

(iii) Previous steps are repeated 1000 times to provide 1000 samples for $\boldsymbol{d}_X^T(\lambda_j), j = 1, \cdots, J$.

(iv)Calculate the value of χ^{2^*} and then compare it with $\chi^2_{1-\alpha}(2T)$.

(v)Previous steps are repeated one thousand times to estimate the level and the power of the test that can be respectively computed by

$$\widehat{\alpha} = \frac{\text{The number of runs for which the value of } \chi^{2^*} \text{ is more than } \chi^2_{1-\alpha}(2T), \text{ under } H_0}{1000},$$

$$\widehat{\pi} = \frac{\text{The number of runs for which the value of } \chi^{2^*} \text{ is more than } \chi^2_{1-\alpha}(2T), \text{ under } H_1}{1000}.$$

Example 1:. *Suppose the time series*

$$X_t^{(i)} = \phi_t^{(i)} X_{t-1}^{(i)} + Z_t^{(i)}, \{Z_t^{(i)}\}\ IIDN(0,1), i = 1, 2, 3,$$

such that $\phi_t^{(i)} = \frac{0.4+\phi^{(i)}\cos\left(\frac{2\pi t}{T}\right)}{2}, \phi^{(1)} = 0.5, \phi^{(2)} = 0.2, 0.5$ and $\phi^{(3)} = 0.5, 0.8.$

Example 2:. *Suppose the time series*

$$X_t = Z_t^{(i)} + \theta_t^{(i)} Z_{t-1}^{(i)}, \{Z_t^{(i)}\}\ IIDN(0,1), i = 1, 2, 3,$$

such that

Table 1 The level and power of the proposed technique for the first example.

		(n_1, n_2, n_3)			
$\phi^{(2)}$	$\phi^{(3)}$	(75, 50, 100)	(100, 75, 150)	(100, 150, 200)	(300, 250, 500)
0.2	0.5	0.832	0.912	0.995	0.999
0.2	0.8	0.864	0.964	0.997	1.000
0.5	0.5	0.051	0.050	0.049	0.049
0.5	0.8	0.812	0.952	0.997	1.000

Table 2 The level and power of the proposed technique for the second example.

		(n_1, n_2, n_3)			
$\theta^{(2)}$	$\theta^{(3)}$	(75, 50, 100)	(100, 75, 150)	(100, 150, 200)	(300, 250, 500)
0.2	0.5	0.802	0.943	0.995	1.000
0.2	0.8	0.865	0.965	0.997	1.000
0.5	0.5	0.051	0.050	0.050	0.050
0.5	0.8	0.843	0.925	0.998	1.000

Table 3 The level and power of the proposed technique for the third example.

		(n_1, n_2, n_3)			
$\theta^{(2)}$	$\phi^{(3)}$	(75, 50, 100)	(100, 75, 150)	(100, 150, 200)	(300, 250, 500)
0.1	0.5	0.811	0.923	0.992	1.000
0.1	0.9	0.821	0.941	0.989	1.000
0.5	0.5	0.051	0.050	0.050	0.049
0.5	0.9	0.834	0.954	0.991	0.999

Table 4 The level and power of the proposed technique for the fourth example.

		(n_1, n_2, n_3)			
$m^{(2)}$	$m^{(3)}$	(75, 50, 100)	(100, 75, 150)	(100, 150, 200)	(300, 250, 500)
0.4	0.5	0.843	0.934	0.999	1.000
0.4	0.8	0.876	0.965	0.998	1.000
0.5	0.5	0.050	0.050	0.049	0.049
0.5	0.8	0.843	0.954	0.998	1.000

$\theta_t^{(i)} = \frac{0.5+\theta^{(i)}\cos\left(\frac{2\pi t}{T}\right)}{2}, \theta^{(1)} = 0.5, \theta^{(2)} = 0.2, 0.5$ and $\theta^{(3)} = 0.5, 0.8.$

Example 3:. *Suppose the time series*

$$X_t^{(i)} - \phi_t^{(i)} X_{t-1}^{(i)} = Z_t^{(i)} + \theta_t^{(i)} Z_{t-1}^{(i)}, \{Z_t^{(i)}\}\ IIDN(0,1), i = 1,2,3,$$

such that

$\phi_t^{(i)} = \frac{0.4+\phi^{(i)}\cos(2\pi t/T)}{2}$, $\theta_t^{(i)} = \frac{0.7+\theta^{(i)}\cos\left(\frac{2\pi t}{T}\right)}{2}, \phi^{(1)} = \theta^{(1)} = 0.5,$ $\phi^{(2)} = 0.5,\ \theta^{(2)} = 0.1, 0.5, \phi^{(3)} = 0.5, 0.9$ and $\theta^{(3)} = 0.5.$

Example 4:. *Suppose the time series*

$$X_t^{(i)} = \left(1 + m^{(i)}\cos(2\pi t/T)\right) Z_t^{(i)}, \{Z_t^{(i)}\}\ IIDN(0,1), i = 1,2,3,$$

where

$m^{(1)} = 0.5,\ m^{(2)} = 0.4, 0.5$ and $m^{(3)} = 0.5, 0.8.$

The estimated $\widehat{\alpha}$ with $T = 2, 3, 4,$ and 5, for Examples 1–4, are summarized in the third rows of Tables 1 to 4, respectively. Since these values are very near to the $\alpha = 0.05$, particularly when (n_1, n_2, n_3) grow, then the introduced method controls the Type I error ($\alpha = 0.05$). Other rows of Tables 1-4 are correspond to the values of $\widehat{\pi}$. These values verify the excellent ability of the introduced technique to discriminate between H_0 and H_1.

Also the Q–Q plots for the values of the test statistic χ^{2^*} against the $\chi^2(2T)$ distribution have been presented in Fig. 1.

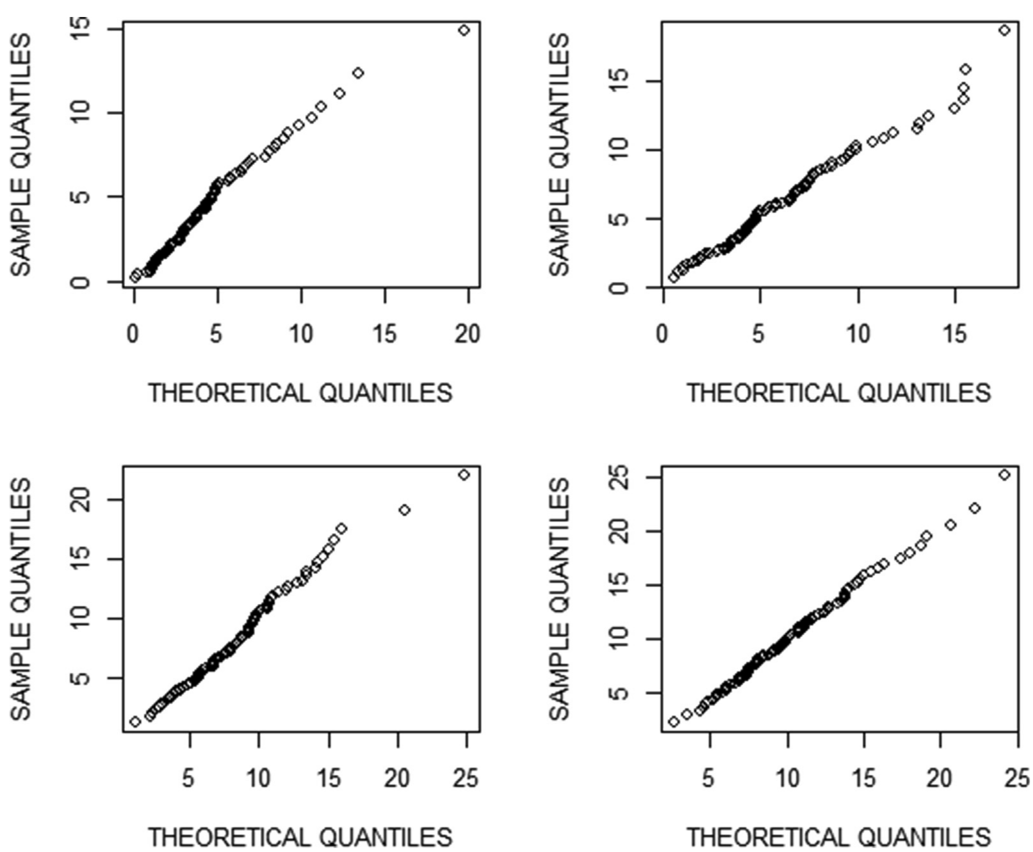

Fig. 1 Q–Q plots for the test statistic χ^{2^*} against the $\chi^2(2T)$ distribution, First row, First column is corresponded to Example 1 *with sample sizes*$(n_1, n_2, n_3) = (100, 50, 75)$. First row, Second column is corresponded to Example 2 *with sample sizes*$(n_1, n_2, n_3) = (150, 75, 100)$. Second row, First column is corresponded to Example 3 *with sample sizes*$(n_1, n_2, n_3) = (200, 150, 100)$. Second row, Second column is corresponded *to* Example 4 *with sample sizes*$(n_1, n_2, n_3) = (5002, 50, 300)$.

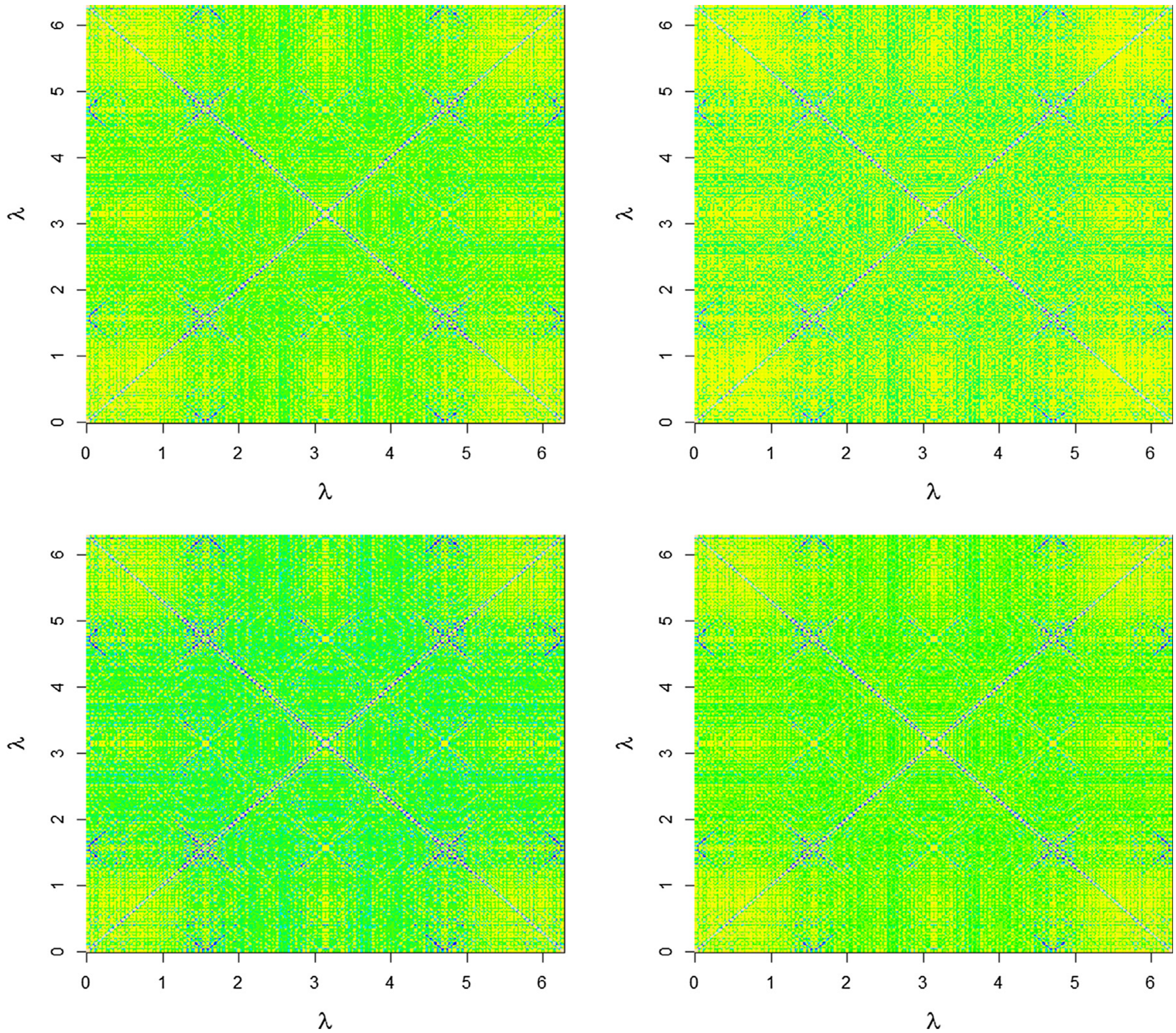

Fig. 2 CSS plot for different sections (Above: Left: Section One, Right: Section Two; Below: Left: Section Three, Right: Section Four).

Table 5 The results of the proposed approach to compare the spectral densities of different sections.

Test Statistic	P-Value
$\chi^{2^*}=8.759$	0.363

As it can be seen, the points are close to strain line and consequently the test statistic χ^{2^*} is asymptotically $\chi^2(2T)$. Therefore, the presented approach performs well in simulation.

5. Real data

In this section, the performance of the proposed technique is examined for real world problem. The considered real dataset is the logarithms of the Real Gross Domestic Product in Germany [28]. We divide whole dataset in four separate sections; Section one: from spring 1960 to winter 1967, Section two: from spring 1968 to winter 1975, Section three: from spring 1976 to winter 1984, and Section four: from spring 1985 to winter 1990. Fig. 2 demonstrates the CSS plots [25] for these sections. These plots, determine a CS-4 time series for all sections that verify the time series given by [28]. Therefore all of sections follow from a CS-4 model.

Table 5 summarizes the results of the given approach to test the hypothesis $H_0: f_1 = \cdots = f_4$. Since the p-value is more than 0.05 ($p = 0.363$), consequently we accept that all sections have similar spectral densities.

6. Conclusion

Comparison of several time series is a main subject in economics, physics, chemistry, signal processing and other scientific fields. Really, the researchers try to compare the stochastic mechanism of some observed time series. The classification, comparison and clustering of two or some processes have been investigated in different time-domain and frequency-domain methods. Most of these approaches can be utilized to the stationary processes or the non-stationary processes which are transformable to stationary processes by using differencing. This paper was devoted to compare the spectral densities of uncorrelated cyclostationary time series. By using the limiting distributions of the discrete Fourier transform, a novel approach was introduced to compare the spectral densities of uncorrelated cyclostationary processes. The performance of the proposed technique was examined by employing numerous simulation and real examples. The presented technique performed well in simulation. This proposed method is also controlled the Type I error and verified the excellent ability to discriminate between H_0 and H_1.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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