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Aerodynamic effects of horizontal-temporal variability of atmospheric air density – A first approach

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As we know, the lift force and airplane velocity depend on moist (real) air density. Accordingly we discussed the influence of mentioned aerodynamic parameters based on horizontal-temporal changing of air density around their mean values. Using small perturbation of mean air pressure and temperature we determined the horizontal-temporal variability of the air density and airplane velocity as True Airspeed (TAS). The 10% influence of air temperature, air pressure and vapor pressure involve about 2.4%, 0.8% and 0.05% air density variability, respectively. The same 10% decreasing in air temperature causes 1.2% positive changing in assumed mean TAS. The short-term variability of moist air density has also been investigated and our results show 3–4% changing in TAS during a strong cold front crossing. On the other hand the climatic variability of atmospheric state variables much higher than 10% thus the long-term (climatic) variation of real atmospheric air density (and TAS of course) has to be very significant variability in time!

1. Introduction

It is well-known that the possibility of aviation is based on appearing the resultant aerodynamic force when the airflow is moving around the wing of an airplane. (We can see the similar effect in case of helicopter rotors, too.) The mentioned force can be divided into two perpendicular components such as lift and drag. The magnitude of lift force (which balances the weight of an airplane during level flight) depends on the velocity and density of moving air. During calculations we often use constant air density but in any cases it is a wrong approach because the density of air has a temporal and spatial variability in the Earth atmosphere. For that very reason it is a good idea to discuss this changing, its effect to magnitude of lift force and related aerodynamic phenomena.

In our work we examined the horizontal-temporal variability of air density and we did not concern with high speed (such as highly compressible) airstream and we also assumed an inviscid flow in the atmosphere.

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2. Aerodynamic forces

If an object is located into airflow we can detect two very important effects so called aerodynamic forces and moments (ANDERSON, 2001). Both of them depend on two basic influences:

- Distribution of pressure over the object's surface
- Distribution of shear stress over the object's surface.

The pressure (p) and shear stress (τ) have the same dimension [Nm^{-2}] and they are acting on the body as it can be seen in Figure 1.

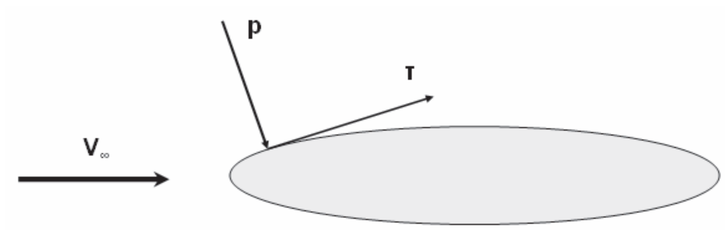

Figure 1. The pressure (p) and shear stress (τ) on the surface of object located in the freestream moving with velocity V_∞ (after ANDERSON, 2001).

The integration of p and τ distributions over the complete body determines the resultant aerodynamic force (R) and moment (M) on the object.

The resultant force (R) can be split into two components:

- Lift component (L) perpendicular to V_∞
- Drag component (D) parallel to V_∞ (Figure 2).

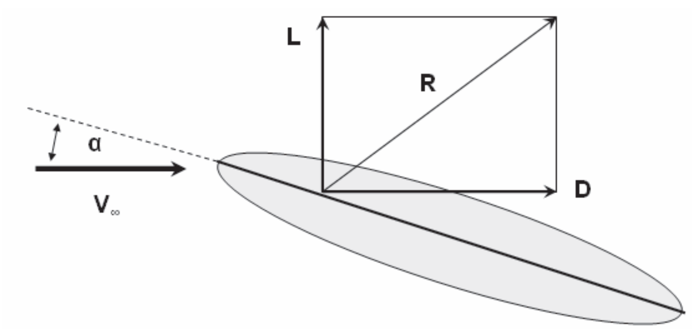

Figure 2. The resultant aerodynamic force and its components: lift (L) and drag (D) force. α is the angle of attack and V_∞ is the freestream velocity (after ANDERSON, 2001).

The role of the lift (L) is fundamental in the aviation: this force balances the effect of gravity (weight) during straight flight.

3. The dynamic pressure, lift and drag coefficients

As we know the streaming air – similar to the moving body – has the kinetic energy-like attribute so we can define it such as dynamic pressure (q_∞ [Nm^{-2}]):

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 \tag{1}$$

where ρ_∞ is the air density and V_∞ is the velocity of freestream.

With the help of dynamic pressure we are able to determine the lift (C_L) and drag coefficients (C_D):

$$C_L = \frac{L}{q_\infty S} \tag{2}$$

$$C_D = \frac{D}{q_\infty S} \tag{3}$$

where L and D are the lift and drag forces, q_∞ is the air density of freestream and S is a reference area (for example in case of an airplane wing S is the planiform area) (ANDERSON, 2001). The mentioned coefficients are dimensionless values and they depend on the angle of attack (α) as it can be seen in Figure 3.

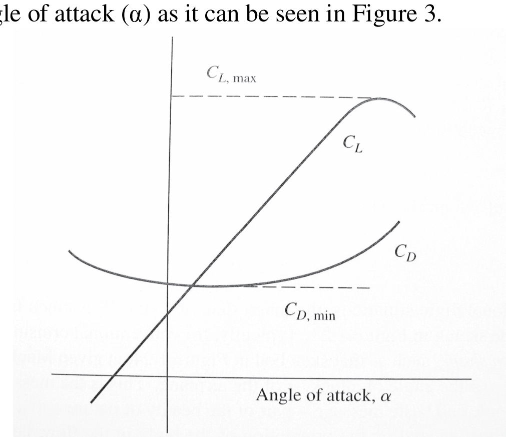

Figure 3. Relationship between lift and drag coefficients and angle of attack (α). The maximum lift ($C_{L,\max}$) and minimum drag coefficients ($C_{D,\min}$) are signed (after ANDERSON, 2001).

At a steady level flight the lift force can be calculated below:

$$L = \frac{1}{2} \cdot S \cdot C_L \cdot \rho_\infty \cdot V_\infty^2 \qquad (4)$$

where (at a given angle of attack) S and C_L are constants so we can write:

$$L = L(\rho_\infty, V_\infty) \qquad (5)$$

This means in a given case that the magnitude of lift force depends on air density and velocity of freestream only. Because we assumed earlier that the airflow is incompressible, in this case the freestream density approximately equals air density we use ρ instead of ρ_∞ hereafter.

4. The airplane velocity and the relationship between IAS, TAS and GS

Let solve the equation (4) for airplane velocity (V_∞):

$$V_\infty = \sqrt{\frac{2W}{\rho S C_L}} \qquad (6)$$

where W is the weight of airplane which substitutes the lift force (L) since the airplane in level flight and in this case L=W. Obtaining the W and S are constants (of course in a real case they are not definitely truth facts) we can see that the airplane velocity depends on air density and angle of attack (because $C_L=C_L(\alpha)$):

$$V_\infty = V_\infty(\rho, \alpha) \qquad (7)$$

As we can see in Figure 3 there is a maximum lift coefficient ($C_{L,\,max}$) in reference to any airfoils thus the stalling velocity (V_{stall}) of flight is determined by the mentioned coefficient:

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L,max}}} \qquad (8)$$

It is important to note that the stalling velocity is a *lowest or minimum speed* which is needed to fly!

The above mentioned velocity means the speed of moving air around the airplane thus it represents approximately the True Airspeed (TAS) in a calm atmosphere so we can write:

$$V_\infty = TAS \qquad (9)$$

On the other hand the pilots usually use the Indicated Airspeed (IAS) instead of TAS during flight because their airspeed indicator shows this value directly. Since the measurement of IAS based on the sensing of difference between static and pitot (impact) pressure of the streaming air if the pilot keeps a constant IAS value during the straight and level flight he really keeps a constant value of qC_L product and thus a constant lift force (of course the pilot can change the C_L, too if it is needed)!

It is clear the applying IAS during flight (with the special regard to final approach and landing) the variability of air density is highly eliminated for the pilots.

However, during a steady flight providing a calm air the decreasing of air density involves the higher TAS keeping the same value of IAS as long as the C_L (or α) is a constant. On the other hand in this case the TAS equals the Groundspeed (GS) (because the winds are eliminated) so both TAS and GS are increasing!

5. The horizontal-temporal variability of moist air density and airplane velocities in consideration of the atmospheric state variables

The atmosphere of the Earth is a mixture of gases and this sphere has a relatively low density. However, in case of moist (real) air the density is not constant in the atmosphere and the relationship between air density and other atmospheric state parameters are the following (*perfect gas low*):

$$\rho = \frac{p}{RT_v}$$

where p, T_v, R and ρ are the air pressure [Pa], virtual temperature [K], gas constant of dry air [287 $J \cdot kg^{-1}K^{-1}$] and air density [$kg \cdot m^{-3}$], respectively (GÖTZ and RÁKÓCZI, 1981). Certainly the virtual temperature signs the appearing of water vapor in the air. Important to know:

$$T_v = (1 + 0.608r)T = T_v(T, r) \geq T$$

where T [K] is the air temperature, r [kg water vapor/kg dry air] is the mixing ratio of moist air. On the other hand the expression of mixing ratio is the following:

$$r = \frac{0.622e}{p - e} = r(e, p)$$

where e and p are the vapor pressure [Pa] and air pressure [Pa] of the moist air (GÖTZ and RÁKÓCZI, 1981). In case of moist (real) atmosphere the air density can be determined with the help of the next equation:

$$\rho = \frac{p}{RT\left(1+\frac{0.378e}{p-e}\right)} = \rho(p,T,e)$$

On the base of this expression we can calculate the variability of moist air density depending on these state variables.

As we saw in the Chapter 4 the TAS (and GS, too) is affected by moist air density. Hence the variability of moist atmospheric state variables influences the value of TAS at a given airplane. If we would like to determine how the TAS changes with the moist air density we apply the next equation (modified after JEREB, 1981):

$$TAS(\rho) = TAS_0 \cdot \sqrt{\frac{\rho_0}{\rho}}$$

where the TAS(ρ), TAS_0, ρ_0 and ρ are True Airspeed at a given moist air density, True Airspeed at the assumed mean moist air density, the assumed moist air density and the given moist air density, respectively (Table 1).

6. Results

Let we assume that the moist atmosphere has the following (mean) values of state variables as we can see in the Table 1. These values have been derived from ICAO Standard Atmosphere (ISA) related to the see level and completed by vapor pressure value. In order to determining the role of mentioned variables in the variability of moist air density we can perturb the given state variable value with 10% of their absolute natural variability in Hungary as long as the other ones have the constant values as it can be seen in Table 1.

Table 1. The (mean) values of state variables in an assumed moist atmosphere

Moist air temperature	Moist air pressure	Vapor pressure	Moist air density
T_0=288.2 K	p_0=101325 Pa	e_0=1000 Pa	ρ_0=1.220202 kgm^{-3}

We calculated the fluctuated moist air density (ρ_{fluct}) using the increasing of 7 K, 800 Pa and 130 Pa values such as the 10% of absolute natural variability of air temperature, air pressure and vapor pressure, respectively. Our results can be found in Table 2.

Table 2. The fluctuated air densities and differences between assumed mean air density and fluctuated ones. In brackets the mentioned differences in the percentage of assumed mean moist air density

Perturbed state variable	Constants	Fluctuated air density	$\Delta\rho = \rho_0 - \rho_{fluct}$
Air temperature (7 K)	p_0, e_0	1.191268 kg·m^{-3}	0.028934 (2.37%)
Air pressure (800 Pa)	T_0, e_0	1.229873 kg·m^{-3}	–0.009671 (0.79%)
Vapor pressure (130 Pa)	T_0, p_0	1.219600 kg·m^{-3}	0.000602 (0.05%)

It follows that the 10% air temperature, air pressure and vapor pressure influences involve about 2.4%, –0.8% and 0.05% air density variability, respectively (Table 2). On the other hand around the mean values of atmospheric state variables the air temperature plays the most important role in the variability of moist air density. We have to underline, this variability is a good approximation in this circumstances only.

Since there is an important exponential relationship between air temperature and vapor pressure thus at a higher air temperature there will be a larger absolute variability of vapor pressure. This problem is not discussed in this work because we are going to analyze it in detail in a future.

After that we calculated TAS(ρ) values in the mentioned different cases (Table 3):

Table 3. The different perturbed TAS values (TAS (ρ))

Perturbed state variable	Fluctuated air density	TAS (ρ)
Air temperature (7 K)	1.191268 kg·m^{-3}	1.012 TAS_0
Air pressure (800 Pa)	1.229873 kg·m^{-3}	0.996 TAS_0
Vapor pressure (130 Pa)	1.219600 kg·m^{-3}	1.000 TAS_0

As it can be seen in Table 3 – obviously similar to the changing of moist air density – the air temperature has the strongest act on TAS. However, this influence seems to be small (10% air temperature decreasing causes 1.2% positive changing in assumed mean TAS) but do not forget that the absolute variability of air temperature is much greater than 7 K in the Carpathian Basin, too! On the other hand the perturbed state variables change in the same time (not separately) so their effects can appear multiple.

For example, the very low air temperature often involves extremely high air pressure over a given location (thermal anticyclonic weather situation) thus the *long-term (climatic) variation* of real atmospheric air density (and TAS and GS of course) have to be very significant variability in time!

However the *short-term variability* of air density can be well observed during a cold front crossing over an aerodrome. A typical strong cold front crossing involves a significant decreasing of temperature and simultaneously increasing of air pressure. The water vapor pressure also decreases but usually later (maybe after several hours because the falling precipitation). As we saw earlier this changing all of the three mentioned meteorological parameters – after the cold front crossing – increase the air density. We can examine the real meteorological circumstances a strong cold front passage (18.07.2009) over Szolnok (LHSN) airport (Figure 4).

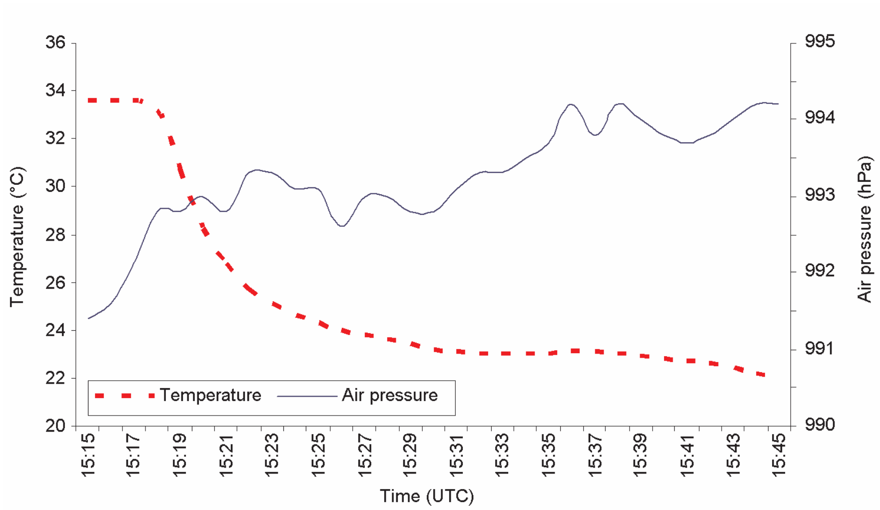

Figure 4. Temperature and air pressure variation during a strong cold front crossing over LHSN airport in 18.07.2009

As we can see in Figure 4 the temperature dropping was 11.5 °C during half an hour and air pressure increased 3.8 hPa simultaneously. Also taking into account of changing amount of water vapor we calculated moist air pressure during mentioned phenomena and our result are seen in Figure 5.

The changing of moist air density was greater than 4% during half an hour and the same variability of moist air density was 8% during last 10 hours after crossing the cold front.

Assuming a steady low-level flight, constant IAS, during the same weather situation the TAS has to be dropped (or jumped) by approximately 3–4% when the airplane crossing the cold frontal plane. This increased (or decreased) TAS value does not seems to be significant but do not forget, the mentioned changing is related to fast horizontal air density variability only! This difference between two TAS values would be important for navigating during a longer flight. The above mentioned 3–4% difference between two TAS values resulted by variability of moist air density alone involves about 15–20 km longitudinal uncertainty in the positioning in case of a two hours flight!

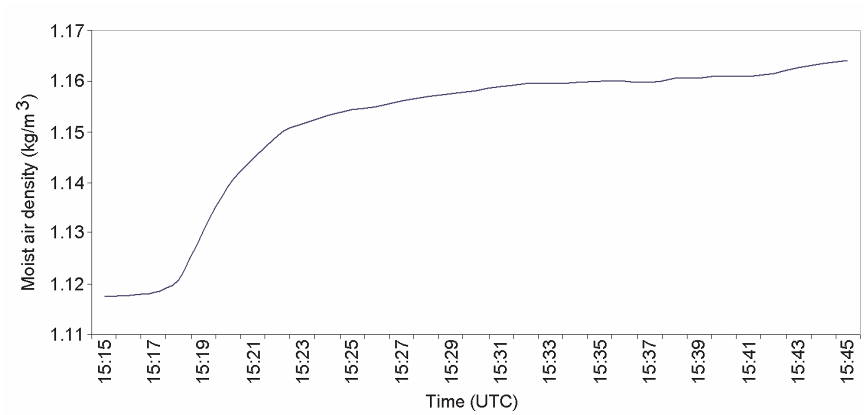

Figure 5. The calculated moist air density variability during a strong cold front crossing over LHSN airport in 18.07.2009

Nevertheless the long-term variability of moist air density are much higher than short-term one thus in our next paper we are going to discuss it in detail.

7. Conclusion

As we showed the values of lift force and airplane velocity depend on the air density. The variability of atmospheric state variables (air temperature, air pressure and water vapor pressure) creates a significant changing in moist air density thus influences airplane velocity, too. However, the variability of atmospheric state variables has a strong connection with the weather and climatic fluctuation as well.

We determined the 10% air temperature, air pressure and vapor pressure influences (around the assumed mean values of state variables) involve about 2.4%, 0.8% and 0.05% air density variability's, respectively.

On the other hand the mentioned influence affects the value of airplane velocity, too. As a result of the same 10% changing of atmospheric state variables the TAS varies about 1, 2% around its assumed mean value. Taking account of the magnitude of real variability of atmospheric state variables the computed variability of air density and TAS will be much higher than presented ones.

A short-term horizontal variability of moist air density during a strong cold front crossing involves appreciatively 3–4% changing in TAS.

Having knowledge of behavior of aerodynamic attributes of airplanes in connection with atmospheric fluctuations is very important in flight safety.

8. References

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