THEORETICAL BACKGROUNDS OF THE RING LASER GYROS

Bertold BÉKÉSI
Senior Lecturer
"Miklós Zrínyi" National Defence University
Faculty of Management and Command
Department of Aircraft Onboard Systems

INTRODUCTION

One of the more promising applications of the laser is as a gyroscope (Heer, 1961; Rosenthal, 1962; Macek and Davis, 1963; McCartney, 1966; Killpatrick, 1967). The laser gyro is an integrating rate gyroscope in the unconventional sence, since it contains no spinning mass. The essential feature of the laser gyro is a ring-type cavity and which the laser radiation traverses a closed path. The laser cavity supports two independent, oppositely directed travelling waves that can oscillate at different frequencies of oscillation of the travelling waves are dependent on the rotation of the cavity with respect to inertial space. A measurement of the rotation of the laser cavity.

From a system point of view, the laser gyro can be considered as just another black box. Power is applied and information is taken out and fed into a computer. This survey article provides further insight into the basic operation of the laser gyro.[5]

Discussion of the laser gyro can be divided into three parts: The basic concept involved in its operation (1), the active laser medium (2), and the cavity (3). This article deals mainly with the basic concept involved in its operation.

The emphasis on the laser material and operation of the laser gyro bring to the surface potential problem areas that must be considered in the design and construction of the laser gyro.

The successful operation of the laser gyro, and its eventual acceptance as a device will be determined mainly by how well and how economically it can be designed and constructed.

PRINCIPLE OF OPERATION

PASSIVE SAGNAC INTERFEROMETER

The principle of operation of the laser gyro is best described by first considering a rotating ring interferometer, as first successfully demonstrated by Sagnac (1913). Since the effect is first order in v/c, classical theory will give the correct answer to first order. Strictly speaking, the special theory of relativity is not applicable, since the light must be considered on a rotating frame. The only rigorously correct theory is the general theory of relativity. However, for conceptual simplicity, the rotating interferometer will be considered, using classical theory.
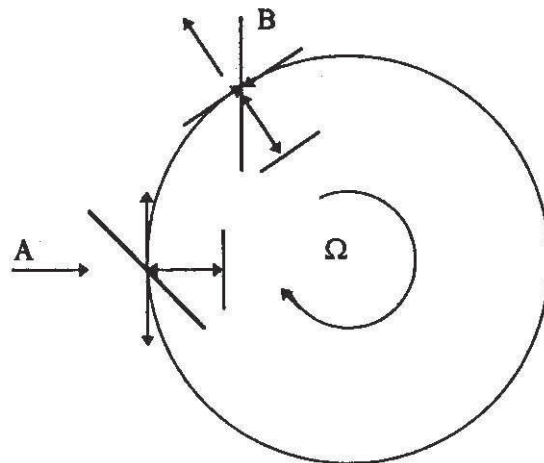

Figure 1.
Circular rotating (Sagnac interferometer)

Figure 1. shows an ideal circular interferometer of radius R [3, 4]. Light enters at point A and is split by the beamsplitter. In this ideal interferometer, the light is constrained to travel along the circumference of the circle. After travelling along the complete path, the light recombines at the original beamsplitter. When the interferometer is stationary, the transit time for the light to make a complete path is the same for both beams and is given by

$$t = \frac{2\pi R}{c} \tag{1}$$

where: c - is the velocity of light

If the interferometer is rotated at a constant speed Ω, the closed path transit time is modified from that given by eq. (1). In fact the closed path transit time becomes different for light travelling with and against the direction of rotation. This occurs because of the fact that during the closed path transit time of the light, the beamsplitter, originally located at point A, moves to point B. Thus, with respect to inertial space, light travelling against and with the direction of rotation must traverse a smaller and greater distance, respectively, than when there is no rotation. Note that the speed of the light is considered to remain invariant. Then the closed path transit time for the light is given by the equations

$$2\pi R \pm X_{\pm} = ct_{\pm}$$
$$X_{\pm} = R\Omega t_{\pm}$$

or

$$t_{\pm} = \frac{2\pi R}{c \mp R\Omega} \tag{2}$$

The upper sign in eq. (2) refers to the light travelling in the direction of rotation and X refers to the inertial space distance between points A and B. Note that eq. (2) can be interpreted in terms of the speed of light being different for the two directions and the path length being the same [1, 5].

The closed-path transit time difference for the light travelling in opposite directions is given by

$$\Delta t = t_{+} - t_{-}$$

and from eq. (2), we find, to first order

$$\Delta t = \frac{4\pi\,\Omega R^2}{c^2} \tag{3}$$

This difference in closed-path transit time for light travelling in opposite directions gives rise to an optical path difference of $c\Delta t$, or from eq. (3),

$$\Delta L = \frac{4\pi\Omega R^2}{c} \tag{4}$$

This is the basic equation for the rotating interferometer. It shows that the optical path difference is proportional to the area enclosed by the light and the rotation speed. Equation (4) does not take into account effects due to the presence of optically refracting materials in the path of the light beams[4, 6, 7,8].

According to the general theory of relativity, a clock travelling on a rotating frame loses synchronization with one located on a stationary frame. This loss of synchronization gives rise to a different closed-path transit time for light travelling in opposite directions on a rotating frame, or

$$\Delta t = \oint 2\Omega R^2 \left[1-\left(\frac{\Omega R}{c}\right)^2\right]^{-1} d\varphi \tag{5}$$

There the integral is taken over a closed contour. Neglecting second-order terms, eq. (5) becomes

$$\Delta t = \left(\frac{2\Omega}{c^2}\right)\oint r^2 d\varphi$$

$$\Delta t = \frac{4A\Omega}{c^2}$$

where: A - is the area enclosed by the light.

Thus eq. (4) can be generalized for an arbitrary cavity configuration as

$$\Delta L = \frac{4A\Omega}{c} \tag{6}$$

From the development eq. (5) the optical path difference given by eq. (6) is independent of the location of the axis of rotation. It should also be noted that a

256

measurement of the optical path difference enables an observer located on the rotating frame to measure the so called "absolute" rotation of his frame.

ACTIVE RING LASER INTERFEROMETER

As discussed in section, the use of a ring resonator in the passive mode allowed the determination of the rotation (with respect to inertial space) of the resonator. The observer is located on the rotating frame. A light source, external to the cavity, is used and the quantity measured is the phase difference arising from the unequal path lengths for light travelling in opposite directions round the rotating cavity [1, 5, 6, 7, 8].

The difficulty in using the Sagnac interferometer as a practical device arises from lack of sensitivity, since the path difference for light travelling in the two directions is much less than a wavelength. The use of a laser as the external light source does not help. However, if the system is made into an active interferometer, the situation changes markedly. The improvement in sensitivity arises from the fact that the laser frequency is dependent on the cavity length.
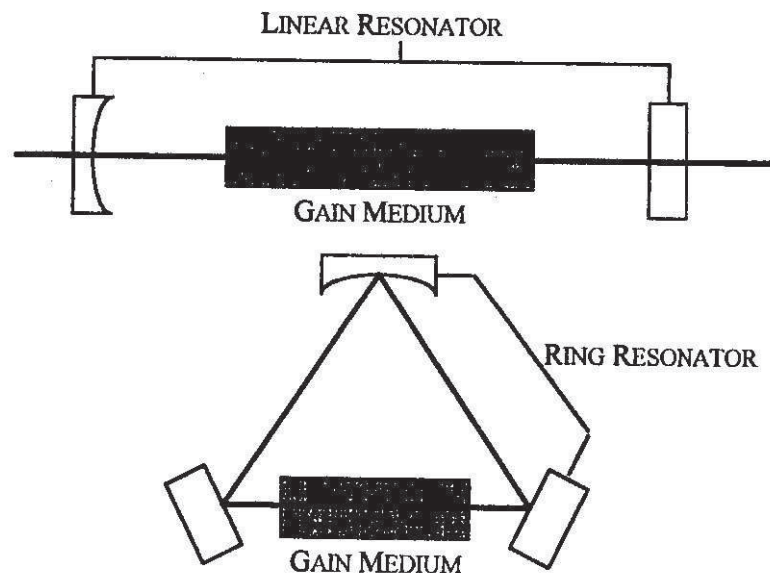

Figure 2. Linear laser and ring laser cavity configuration

Figure 2. is a schematic of a linear laser with two mirrors separated by a distance l and a ring laser with total perimeter L. In both cases the frequency of

oscillation condition for the lowest order transverse mode is that the optical cavity length encloses an integral number of wavelengths. For the linear laser the cavity modes consist of two oppositely directed travelling waves, which compose a standing wave. The amplitudes and frequencies of the traveling waves are constrained to be equal. In the ring laser each cavity mode also consists of two oppositely directed travelling waves. In this case the oppositely directed waves are independent, in the sense that they can oscillate with different amplitudes and frequencies. In fact, whether or not the ring laser could sustain stable oscillations in both directions was not answered until it was actually archived (Macek and Davis, 1963).

If m represents the mode number (typically on the order of 10^5 - 10^6), the oscillation condition can be expressed as

$$m\lambda_{\pm} = L_{\pm}$$

or

$$v_{\pm} = \frac{mc}{L_{\pm}} \tag{7}$$

where: $v_{\pm}$ represents the frequency of the wave which sees the cavity as being of length $L_{\pm}$ respectively.

Thus small changes in the path length result in a small frequency change given by

$$\frac{\Delta v}{v} = \frac{\Delta L}{L} \tag{8}$$

Due to the high frequency in the optical region (10^{14} Hz), small length changes can result in large measurable frequency differences. Letting ΔL, as given by eq. (6), represent the differential cavity length for the oppositely directed waves, the beat frequency can be found from eq. (8) as

$$\Delta v = \frac{4A\Omega}{L\lambda} \tag{9}$$

For a rotation of 10 deg/hr and for a ring laser with an equilateral triangular cavity of 13.2 cm per side operating at a wavelength of 0.633 μm, eq. (9) gives a beat frequency of 5.9 Hz. Using heterodyne techniques, this beat frequency is
258

readily measurable, although it amounts to only 10^{-14} of the value of the optical frequency.

From eq. (7) it can be seen that thermal and mechanical instabilities can cause frequency variations far greater than the rotational beat frequency. Hence for the operation of the ring laser as a rotation sensor (laser gyro) it is necessary for both beams to physically occupy the same cavity.

READOUT IN THE LASER GYRO

In the laser gyro, rotation information is obtained by monitoring the oppositely directed waves. In the ideal case of a uniformly rotating laser, the frequencies of the waves are slightly different; the difference being given by eq. (9). Thus a direct measurement of the beat frequency gives a number proportional to the rotation rate [5, 6].
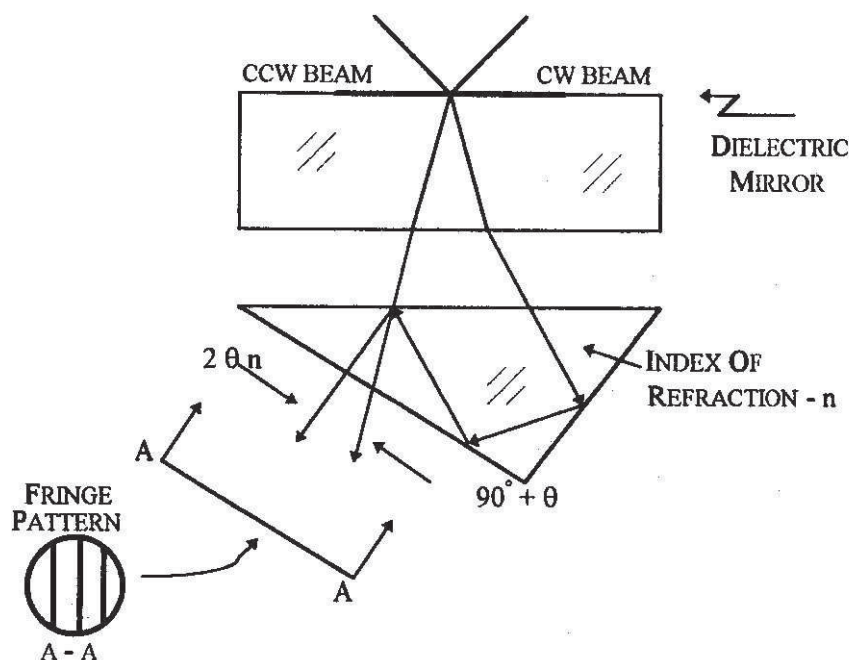

Figure 3. Readout technique in the laser gyro

Figure 3. shows a method of combining the oppositely directed beams to obtain readout. A small percentage (typically less than 0.1%) of the energy of both beams is transmitted through one of the dielectric coated mirrors. The beams

are made approximately collinear by a 90 degree corner prism to form a fringe pattern. The prism can be directly mounted to the mirror to minimize vibrations. Typically, a semitransparent coated mirror is used to match intensities.

The fringes are a measure of the instantaneous phase difference between the oppositely directed beams. For the case when the intensities are matched and the beams are nearly collinear (angular divergence of ε), the fringe pattern is given by

$$I = I_0 \left[1 + \cos\left(\frac{2\pi\varepsilon x}{\lambda} + \Delta\omega t + \varphi \right) \right] \qquad (10)$$

where: $\Delta\omega$ - is the angular beat frequency

φ - is same arbitrary angle

Thus when the laser is not rotating, $\Delta\omega = 0$, and the fringe pattern is stationary. When the laser is rotated, the fringe pattern moves at the beat frequency rate. The fringe spacing is given by $\dfrac{\lambda}{\varepsilon}$. For a parallel substrate ε is given by

$$\varepsilon = 2n\theta$$

where: n - is the index of refraction of the prism

θ - is the deviation of the prism angle from 90 degrees.

For a prism angle deviation of 15 arc sec and for the 0.633 μm He-Ne transition, the fringe spacing is 3 mm. Thus by the use of a detector whose dimensions are much smaller than the fringe spacing, a measurement of the rotation rate can be made by simply recording the rate at which intensity maximum moves past the detector. From eq. (10) it can be seen that the sense of rotation determines the direction in which the fringe pattern moves. Thus by using two detectors spaced 90 degrees (a quarter fringe) apart and a logic circuit, both positive and negative counts can be accumulated to give rotation sense.

It should be noted that with this type of readout, the laser gyro is inherently an integrating rate gyro with a digital output. This can be seen from eq. (9), where a time integration gives

$$N = \left(\frac{4A}{\lambda L} \right) \theta \qquad (11)$$

where

$$N = \int_0^t \Delta v \, dt \, , \qquad \theta = \int_0^t \Omega \, dt$$

CONCLUSIONS

1. The interference picture, formed by recombination of two light beams, travelling in opposite directions, originated from a light source on a rotating frame, is proportional to the rotating speed of the frame;

2. The ring interferometer is suitable for accurate measuring of low rotating speeds.

REFERENCES

[1] Байбородин Ю. В.: Основы лазерной техники, Высшая школа, Киев, 1988.

[2] DR. BENOLÁK Kálmán: A fény, Műszaki Könyvkiadó, Budapest, 1981.

[3] DR. CSILLAG László - DR. KROÓ Róbert: A lézer titkai, Kozmosz Könyvek, 1987.

[4] FOWLER R. G. - MEYER D. I. : Fizika mérnököknek, Műszaki Könyvkiadó, Budapest, 1963.

[5] ED BY MONTE Ross: Laser Aplications, Academic Press, New York and London, 1971.

[6] Д. С. ПЕЛЬПОР: Гироскопические системы, Высшая школа, Москва, 1988.

[7] Д. С ПЕЛЬПОР, Ю. А. ОСОРКИН, Е. Р. РАХТЕЕНКО: Гироскопические прибори систем ориентации и стабилизации, Машиностроение, Москва, 1977.

[8] SZABOLCSI, Róbert: Navigációs rendszerek, Szolnoki Repülőtiszti Főiskola, Főiskolai jegyzet, Szolnok, 1994.